7 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- exponential function, *p. 478*
- exponential growth function, *p. 478*
- growth factor, *p. 478*
- asymptote, *p. 478*
- exponential decay function, *p. 486*
- decay factor, *p. 486*
- natural base *e*, *p. 492*
- logarithm of *y* with base *b*, *p. 499*
- common logarithm, *p. 500*
- natural logarithm, *p. 500*
- exponential equation, *p. 515*
- logarithmic equation, *p. 517*

VOCABULARY EXERCISES

1. What is the asymptote of the graph of the function $y = -2\left(\frac{1}{4}\right)^{x+1} + 5$? $y = 5$

2. Identify the decay factor in the model $y = 7.2(0.89)^x$. **0.89**

3. **WRITING** *Explain* the meaning of $\log_b y$. ***Sample answer:*** $\log_b y = x$ **if and only if** $b^x = y$.

4. Copy and complete: A logarithm with base *e* is called a(n) __?__ logarithm. **natural**

5. Is $y = (1.4)^x$ an *exponential function* or a *power function*? *Explain.*
Exponential function. *Sample answer:* The variable is in the exponent.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 7.

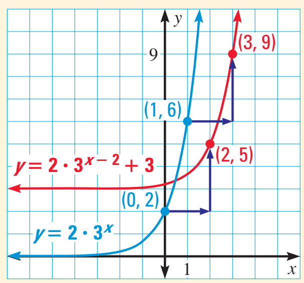
7.1 Graph Exponential Growth Functions *pp. 478–485*

EXAMPLE

Graph $y = 2 \cdot 3^{x-2} + 3$. State the domain and range.

Begin by sketching the graph of $y = 2 \cdot 3^x$, which passes through (0, 2) and (1, 6). Then translate the graph right 2 units and up 3 units. Notice that the translated graph passes through (2, 5) and (3, 9).

The graph's asymptote is the line $y = 3$. The domain is all real numbers, and the range is $y > 3$.



EXERCISES

EXAMPLES 1, 2, 3, and 5 on pp. 478–481 for Exs. 6–9

Graph the function. State the domain and range. **6–8. See margin.**

6. $y = 5^x$

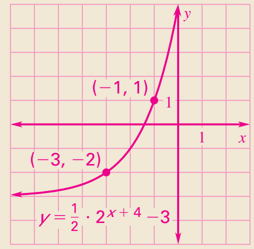
7. $y = 3(2.5)^x$

8. $f(x) = -3 \cdot 4^{x+1} - 2$

9. **FINANCE** You deposit $1500 in an account that pays 7% annual interest compounded daily. Find the balance after 2 years. **$1725.39**

Extra Example 7.1

Graph $y = \frac{1}{2} \cdot 2^{x+4} - 3$. State the domain and range. **domain: all real numbers; range: $y > -3$**



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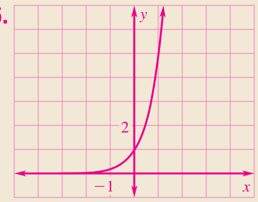
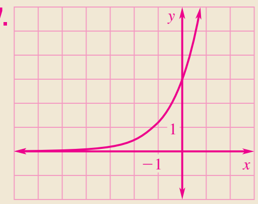


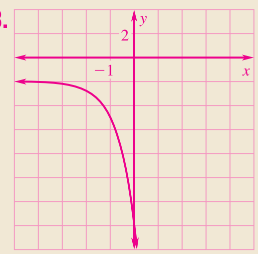
domain: all real numbers, range: $y > 0$

7.



domain: all real numbers, range: $y > 0$

8.

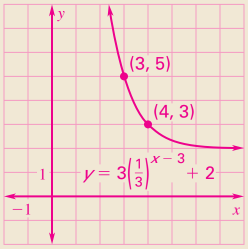


domain: all real numbers, range: $y < -2$

7 CHAPTER REVIEW

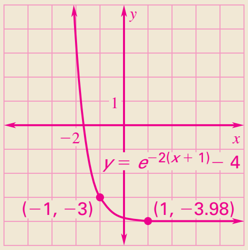
Extra Example 7.2

Graph $y = 3\left(\frac{1}{3}\right)^{x-3} + 2$. State the domain and range. **domain: all real numbers; range: $y > 2$**



Extra Example 7.3

Graph $y = e^{-2(x+1)} - 4$. State the domain and range. **domain: all real numbers; range: $y > -4$**



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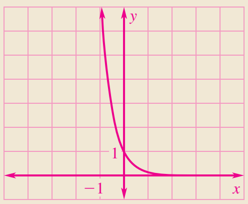
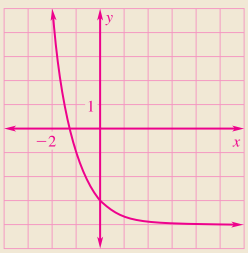


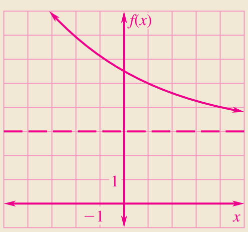
domain: all real numbers, range: $y > 0$

11.



domain: all real numbers, range: $y > -4$

12.



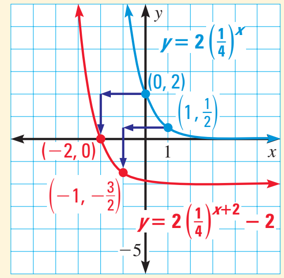
domain: all real numbers, range: $y > 3$

7.2 Graph Exponential Decay Functions *pp. 486–491*

EXAMPLE

Graph $y = 2\left(\frac{1}{4}\right)^{x+2} - 2$. State the domain and range.

Begin by sketching the graph of $y = 2\left(\frac{1}{4}\right)^x$, which passes through (0, 2) and $\left(1, \frac{1}{2}\right)$. Then translate the graph left 2 units and down 2 units. Notice that the translated graph passes through (−2, 0) and $\left(-1, -\frac{3}{2}\right)$.



The graph's asymptote is the line $y = -2$. The domain is all real numbers, and the range is $y > -2$.

EXAMPLES 1, 2, and 3 on pp. 486–487 for Exs. 10–12

EXERCISES

Graph the function. State the domain and range. **10–12. See margin.**

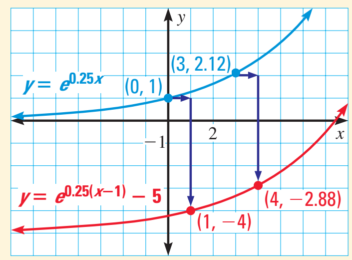
10. $y = \left(\frac{1}{8}\right)^x$ **11.** $y = \left(\frac{1}{3}\right)^x - 4$ **12.** $f(x) = 2(0.8)^{x-1} + 3$

7.3 Use Functions Involving *e* *pp. 492–498*

EXAMPLE

Graph $y = e^{0.25(x-1)} - 5$. State the domain and range.

Because $a = 1$ is positive and $r = 0.25$ is positive, the function is an exponential growth function. Begin by sketching the graph of $y = e^{0.25x}$. Translate the graph right 1 unit and down 5 units.



The domain is all real numbers, and the range is $y > -5$.

EXAMPLES 3 and 5 on pp. 493–495 for Exs. 13–16

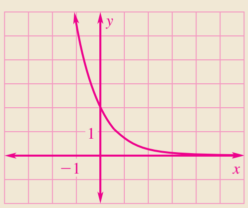
EXERCISES

Graph the function. State the domain and range. **13–15. See margin.**

13. $y = 2e^{-x}$ **14.** $y = e^{x-2}$ **15.** $f(x) = e^{-0.4(x+2)} + 6$

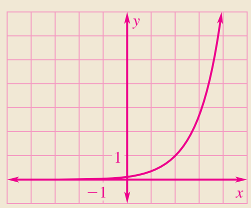
16. PHYSIOLOGY Nitrogen-13 is a radioactive isotope of nitrogen used in a physiological test called positron emission tomograph (PET). A typical PET scan begins with 6.9 picograms of nitrogen-13 (1 picogram = 10^{-12} grams). The number N of picograms of nitrogen-13 remaining after t minutes can be modeled by $N = 6.9e^{-0.0695t}$. How many picograms of nitrogen-13 remain after 10 minutes? **about 3.44 picograms**

13.



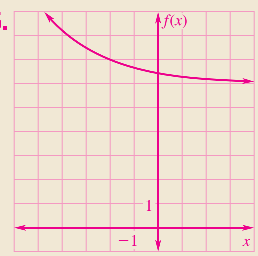
domain: all real numbers, range: $y > 0$

14.



domain: all real numbers, range: $y > 0$

15.



domain: all real numbers, range: $y > 6$

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Chapter Review Practice

7.4 Find Logarithms and Graph Logarithmic Functions *pp. 499–505*

EXAMPLE

Evaluate the logarithm.

a. $\log_5 625$ **b.** $\log 0.001$ **c.** $\log_{125} 5$ **d.** $\log_2 \frac{1}{64}$

To help you find the value of $\log_b y$, ask yourself what power of b gives you y.

a. 5 to what power gives 625?
$5^4 = 625$, so $\log_5 625 = 4$.

b. 10 to what power gives 0.001?
$10^{-3} = 0.001$, so $\log 0.001 = -3$.

c. 125 to what power gives 5?
$125^{1/3} = 5$, so $\log_{125} 5 = \frac{1}{3}$.

d. 2 to what power gives $\frac{1}{64}$?
$2^{-6} = \frac{1}{64}$, so $\log_2 \frac{1}{64} = -6$.

EXERCISES

EXAMPLES 2, 4, 7, and 8 on pp. 500–503 for Exs. 17–24

Evaluate the logarithm without using a calculator.

17. $\log_3 243$ **5** **18.** $\log_7 1$ **0** **19.** $\log_{1/6} 216$ **−3** **20.** $\log_{125} \frac{1}{5}$ $-\frac{1}{3}$

Graph the function. State the domain and range. **21–23. See margin.**

21. $y = \log_{1/6} x$ **22.** $y = \log_3 x - 4$ **23.** $f(x) = \ln (x - 1) + 3$

24. BIOLOGY Researchers have found that after 25 years of age, the average size of the pupil in a person's eye decreases. The relationship between pupil diameter d (in millimeters) and age a (in years) can be modeled by $d = -2.1158 \ln a + 13.669$. What is the average diameter of a pupil for a person 25 years old? 50 years old? **about 6.86 mm; about 5.39 mm**

7.5 Apply Properties of Logarithms *pp. 507–513*

EXAMPLES

Expand the expression.

$$\log_5 \frac{6x}{y^3} = \log_5 6x - \log_5 y^3$$
$$= \log_5 6 + \log_5 x - \log_5 y^3$$
$$= \log_5 6 + \log_5 x - 3\log_5 y$$

Condense the expression.

$$3\log_3 8 - \log_3 16 = \log_3 8^3 - \log_3 16$$
$$= \log_3 \frac{8^3}{16}$$
$$= \log_3 32$$

EXERCISES

EXAMPLES 2 and 3 on p. 508 for Exs. 25–31

Expand the expression.

25. $\log_8 3xy$ $\log_8 3 + \log_8 x + \log_8 y$ **26.** $\ln 10x^3y$ $\ln 10 + 3\ln x + \ln y$ **27.** $\log \frac{8}{y^4}$ $\log 8 - 4\log y$ **28.** $\ln \frac{3y}{x^5}$ $\ln 3 + \ln y - 5\ln x$

Condense the expression.

29. $3\log_7 4 + \log_7 6$ $\log_7 384$ **30.** $\ln 12 - 2\ln x$ $\ln \frac{12}{x^2}$ **31.** $2\ln 3 + 5\ln 2 - \ln 8$ $\ln 36$

Extra Example 7.4

Evaluate the logarithm.

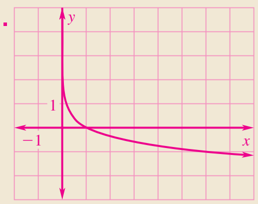
a. $\log_2 128$ **7**
b. $\log 100{,}000$ **5**
c. $\log_{1024} 4$ $\frac{1}{5}$
d. $\log_5 \frac{1}{625}$ **−4**

Extra Examples 7.5

1. Expand $\log_6 \frac{5\sqrt[3]{x}}{y^4}$.
$\log_6 5 + \frac{1}{3}\log_6 x - 4\log_6 y$

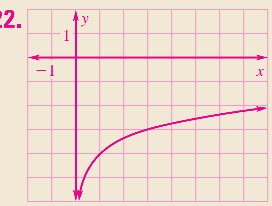
2. Condense $3\ln 4 + \ln 9 - \ln 3$.
$\ln 24$

21.



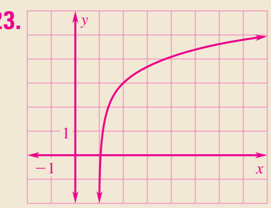
domain: $x > 0$,
range: all real numbers

22.



domain: $x > 0$,
range: all real numbers

23.



domain: $x > 0$,
range: all real numbers

Extra Example 7.6

Solve the equation.

a. $18^x = 10$ about 0.797

b. $\log_2 2x + \log_2 (x + 4) = 6$ 4

Extra Example 7.7

Write an exponential function $y = ab^x$ whose graph passes through (2, 45) and (4, 405). $y = 5 \cdot 3^x$

7 CHAPTER REVIEW

7.6 Solve Exponential and Logarithmic Equations

pp. 515–522

EXAMPLE

Solve the equation.

a.

$$7^x = 12$$
$$\log_7 7^x = \log_7 12$$
$$x = \log_7 12$$
$$x = \frac{\log 12}{\log 7} \approx 1.277$$

b.

$$\log_2 (3x - 7) = 5$$
$$2^{\log_2 (3x - 7)} = 2^5$$
$$3x - 7 = 32$$
$$x = 13$$

EXAMPLES 2, 5, and 6 on pp. 516–518 for Exs. 32–34

EXERCISES

Solve the equation. Check for extraneous solutions.

32. $5^x = 32$ about 2.153

33. $\log_3 (2x - 5) = 2$ 7

34. $\ln x + \ln (x + 2) = 3$ about 3.592

7.7 Write and Apply Exponential and Power Functions

pp. 529–536

EXAMPLE

Write an exponential function $y = ab^x$ whose graph passes through (−1, 2) and (3, 32).

Substitute the coordinates of the two given points into $y = ab^x$.

$2 = ab^{-1}$ Substitute 2 for y and −1 for x.

$32 = ab^3$ Substitute 32 for y and 3 for x.

Solve for a in the first equation to obtain $a = 2b$, and substitute this expression for a in the second equation.

$32 = (2b)b^3$ Substitute $2b$ for a in second equation.

$32 = 2b^4$ Product of powers property

$16 = b^4$ Divide each side by 2.

$2 = b$ Take the positive fourth root because $b > 0$.

Because $b = 2$, it follows that $a = 2(2) = 4$. So, $y = 4 \cdot 2^x$.

EXAMPLES 1 and 5 on pp. 529–532 for Exs. 35–38

EXERCISES

Write an exponential function $y = ab^x$ whose graph passes through the points.

35. (3, 8), (5, 2) $y = 64\left(\frac{1}{2}\right)^x$

36. (−2, 2), (1, 0.25) $y = \frac{1}{2}\left(\frac{1}{2}\right)^x$

37. (2, 9), (4, 324) $y = \frac{1}{4} \cdot 6^x$

38. SPORTING GOODS A store begins selling a new type of basketball shoe. The table shows sales of the shoe over time. Find a power model for the data. $y = 28.0x^{0.750}$

Week, x	1	2	3	4	5	6
Pairs sold, y	28	47	64	79	94	107

Chapter Test, p. 543

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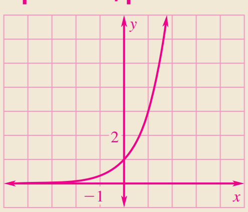

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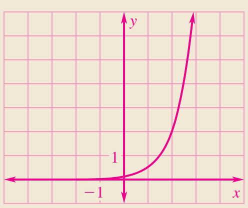

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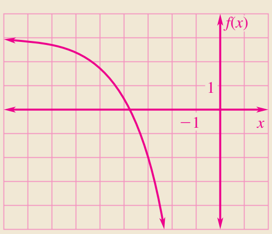

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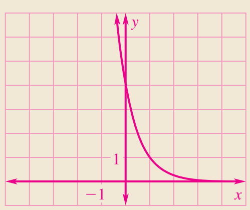

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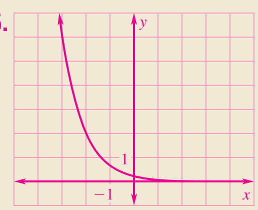

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